

Examples Chapter Transfer Phenomena

Example 1: Cell concentration in aerobic culture

A strain of *Azotobacter vinelandii* is cultured in a 15m^3 stirred Fermenter for alginate production. Under current operating conditions $k_L a$ is 0.17 s^{-1} . Oxygen solubility in the broth is approx. $8 \times 10^{-3}\text{ kg m}^{-3}$.

- The specific rate of oxygen uptake is $12.5\text{ mmol g}^{-1}\text{ h}^{-1}$. What is the maximum possible cell concentration?
- The bacteria suffer growth inhibition after copper sulphate is accidentally added to the fermentation broth. This causes a reduction in oxygen uptake rate to $3\text{ mmol g}^{-1}\text{ h}^{-1}$. What maximum cell concentration can now be supported by the fermenter?

Solution

A strain of *Azotobacter vinelandii* is cultured in a 15 m^3 stirred fermenter for alginate production. Under current operating conditions $k_L a$ is 0.17 s^{-1} . Oxygen solubility in the broth is approximately $8 \times 10^{-3} \text{ kg m}^{-3}$.

- The specific rate of oxygen uptake is $12.5 \text{ mmol g}^{-1} \text{ h}^{-1}$. What is the maximum possible cell concentration?
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Solution:

- From

$$x_{\max} = \frac{(0.17 \text{ s}^{-1})(8 \times 10^{-3} \text{ kg m}^{-3})}{\frac{12.5 \text{ mmol}}{\text{g h}} \cdot \left| \frac{1 \text{ h}}{3600 \text{ s}} \right| \cdot \left| \frac{1 \text{ gmol}}{1000 \text{ mmol}} \right| \cdot \left| \frac{32 \text{ g}}{1 \text{ gmol}} \right| \cdot \left| \frac{1 \text{ kg}}{1000 \text{ g}} \right|}$$
$$= 1.2 \times 10^4 \text{ g m}^{-3} = 12 \text{ g l}^{-1}.$$

- Assume that addition of copper sulphate does not affect C_{AL}^* or $k_L a$. If q_O is reduced by a factor of $12.5/3 = 4.167$, x_{\max} is increased to:

$$x_{\max} = 4.167 (12 \text{ g l}^{-1}) = 50 \text{ g l}^{-1}.$$

To achieve the calculated cell densities all other conditions must be favourable, e.g. sufficient substrate and time must be provided.

Example 2: Specific oxygen uptake in *E.coli* culture

It is assumed, that the specific oxygen uptake rate (qO_2) of *E. coli* is $5.0 \text{ mmol g}^{-1} \text{ h}^{-1}$. Which cell concentration X can be reached in a laboratory reactor with a $k_L a$ of 25 h^{-1} . When $C_L = 10 \% C^*$. and for the medium at 37°C is $C^* = 0.17 \text{ mmol L}^{-1}$

Solution:

$$\text{OTR} = x qO_2$$

with $C_L = 0.1 C^*$

$$\text{OTR} = 25 (0.17 - 0.017) \text{ mmol L}^{-1} \text{ h}^{-1} = 3.8 \text{ mmol L}^{-1} \text{ h}^{-1}$$

$$\text{And } X_{\max} = 3.8 / 5.0 = 0.76 \text{ g L}^{-1}$$

Example 3:

1. Estimate how fast the dissolved oxygen concentration is consumed in a bioreactor with K_{La} 1000 h⁻¹, containing a 10 g/L culture growing with $\mu = 0.5$ h⁻¹ if the aeration is interrupted.

First calculate the quasi-steady state oxygen concentration. Assume $Y_{X/O} = 1$ g/g and the oxygen solubility in the medium equilibrium with air $C^* = 7$ mg/L

Solution:

$$\frac{dC}{dt} = K_{La} (C^* - C) - q_O X$$

$$\frac{dC}{dt} \ll q_O X \text{ and } q_O = \mu / Y_{X/O} \rightarrow C = 0.002$$

$$\text{time} = \frac{C}{r_O} \text{ hrs} = \frac{0.002 * 3600}{0.5/1 * 10} = 1.4 \text{ sec}$$

This calculation assumes that the consumption rate is constant also at very low oxygen concentration.

Solution:

$$\frac{dC}{dt} = K_L a (C^* - C) - q_O X$$

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Because of quasi ss: $Dc/dt = 0 \rightarrow k_{la} (c^* - c) = q_O X$

$$\rightarrow 1000 /h (0.007 \text{ g/L} - ?) = q_O 10 \text{ g/L}$$

$$q_O = \mu e / Y_{XO}$$

$$1000 (0.007 - ?) = 0.5/1 . 10 \rightarrow C = 0.002$$

$$dc/dt = r_O \rightarrow \text{time} = dc / r_O = dc / \mu e (Y_{XO})^{-1} X$$

$$\text{Or } dO/dt = q_O X \rightarrow dt = dO / q_O X$$

Help: Oxygen is a substrate $S = O$

$$(13) \quad Y_{X/S} = \frac{x - x_0}{s_0 - s} = \frac{\Delta x}{\Delta s}$$

$$(14) \quad q_s = \frac{ds}{dt} \cdot \frac{1}{x}$$

$$(15) \quad q_s = \frac{\mu}{Y_{X/S}}$$

$$(16) \quad -\frac{ds}{dt} = q_s \cdot X$$

$Y_{X/S}$ = growth yield

q_s = specific substrate consumption rate

