

# Examples Chapter Transfer Phenomena

# Example 1: Cell concentration in aerobic culture

A strain of *Azotobacter vinelandii* is cultured in a 15m<sup>3</sup> stirred Fermenter for alginate production. Under current operating conditions  $k_L a$  is 0.17 s<sup>-1</sup>. Oxygen solubility in the broth is approx. 8 x 10<sup>-3</sup> kg m<sup>-3</sup>.

- a) The specific rate of oxygen uptake is 12.5 mmol g<sup>-1</sup> h<sup>-1</sup>. What is the maximum possible cell concentration?
- b) The bacteria suffer growth inhibition after copper sulphate is accidentally added to the fermentation broth. This causes a reduction in oxygen uptake rate to 3 mmol g<sup>-1</sup> h<sup>-1</sup>. What maximum cell concentration can now be supported by the fermenter?

# Solution

A strain of *Azotobacter vinelandii* is cultured in a  $15 \text{ m}^3$  stirred fermenter for alginate production. Under current operating conditions  $k_L a$  is  $0.17 \text{ s}^{-1}$ . Oxygen solubility in the broth is approximately  $8 \times 10^{-3} \text{ kg m}^{-3}$ .

- The specific rate of oxygen uptake is  $12.5 \text{ mmol g}^{-1} \text{ h}^{-1}$ . What is the maximum possible cell concentration?
- The bacteria suffer growth inhibition after copper sulphate is accidentally added to the fermentation broth. This causes a reduction in oxygen uptake rate to  $3 \text{ mmol g}^{-1} \text{ h}^{-1}$ . What maximum cell concentration can now be supported by the fermenter?

*Solution:*

- From

$$x_{\max} = \frac{(0.17 \text{ s}^{-1}) (8 \times 10^{-3} \text{ kg m}^{-3})}{\frac{12.5 \text{ mmol}}{\text{g h}} \cdot \left| \frac{1 \text{ h}}{3600 \text{ s}} \right| \cdot \left| \frac{1 \text{ gmol}}{1000 \text{ mmol}} \right| \cdot \left| \frac{32 \text{ g}}{1 \text{ gmol}} \right| \cdot \left| \frac{1 \text{ kg}}{1000 \text{ g}} \right|}$$

$$= 1.2 \times 10^4 \text{ g m}^{-3} = 12 \text{ g l}^{-1}.$$

- Assume that addition of copper sulphate does not affect  $C_{\text{AL}}^*$  or  $k_L a$ . If  $q_{\text{O}}$  is reduced by a factor of  $12.5/3 = 4.167$ ,  $x_{\max}$  is increased to:

$$x_{\max} = 4.167 (12 \text{ g l}^{-1}) = 50 \text{ g l}^{-1}.$$

To achieve the calculated cell densities all other conditions must be favourable, e.g. sufficient substrate and time must be provided.

## Example 2: Specific oxygen uptake in *E.coli* culture

It is assumed, that the specific oxygen uptake rate ( $q_{O_2}$ ) of *E. coli* is  $5.0 \text{ mmol g}^{-1} \text{ h}^{-1}$ . Which cell concentration  $X$  can be reached in a laboratory reactor with a  $k_L a$  of  $25 \text{ h}^{-1}$ . When  $C_L = 10 \% C^*$ . and for the medium at  $37^\circ \text{C}$  is  $C^* = 0.17 \text{ mmol L}^{-1}$

Solution:

$$\begin{aligned} \text{OTR} &= X q_{O_2} \\ \text{with } C_L &= 0.1 C^* \end{aligned}$$

$$\text{OTR} = 25 (0.17 - 0.017) \text{ mmol L}^{-1} \text{ h}^{-1} = 3.8 \text{ mmol L}^{-1} \text{ h}^{-1}$$

$$\text{And } X_{\max} = 3.8 / 5.0 = 0.76 \text{ g L}^{-1}$$

## Example 3:

1. Estimate how fast the dissolved oxygen concentration is consumed in a bioreactor with  $K_La$  1000  $\text{h}^{-1}$ , containing a 10 g/L culture growing with  $\mu = 0.5 \text{ h}^{-1}$  if the aeration is interrupted.

First calculate the quasi-steady state oxygen concentration. Assume  $Y_{X/O} = 1 \text{ g/g}$  and the oxygen solubility in the medium equilibrium with air  $C^* = 7 \text{ mg/L}$

Solution:

$$\frac{dC}{dt} = K_La (C^* - C) - q_O X$$

$$\frac{dC}{dt} \ll q_O X \text{ and } q_O = \mu/Y_{X/O} \longrightarrow C = 0.002$$

$$\text{time} = \frac{C}{r_O} \text{ hrs} = \frac{0.002 * 3600}{0.5/1 * 10} = 1.4 \text{ sec}$$

This calculation assumes that the consumption rate is constant also at very low oxygen concentration.

Solution:

$$\frac{dC}{dt} = K_L a (C^* - C) - q_O X$$

$$\frac{dC}{dt} \ll q_O X \text{ and } q_O = \mu / Y_{XO} \rightarrow C = 0.002$$

$$time = \frac{C}{r_O} \text{ hrs} = \frac{0.002 * 3600}{0.5/1 * 10} = 1.4 \text{ sec}$$

This calculation assumes that the consumption rate is constant also at very low oxygen concentration.

Because of quasi ss:  $Dc/dt = 0 \rightarrow k_L a (c^* - c) = q_{O2} X$

$$\rightarrow 1000 / h (0.007 \text{ g/L} - ?) = q_{O2} 10 \text{ g/L}$$

$$q_{O2} = \mu / Y_{XO}$$

$$1000 (0.007 - ?) = 0.5/1 \cdot 10 \rightarrow C = 0.002$$

$$dc/dt = r_O \rightarrow time = dc / r_O = dc / \mu (Y_{XO})^{-1} X$$

$$\text{Or } dO/dt = q_O X \rightarrow dt = dO / q_O X$$

# Help: Oxygen is a substrate $S = O$

$$(13) \quad Y_{X/S} = \frac{x - x_0}{s_0 - s} = \frac{\Delta x}{\Delta s}$$

$$(14) \quad q_s = \frac{ds}{dt} \cdot \frac{1}{x}$$

$$(15) \quad q_s = \frac{\mu}{Y_{X/S}}$$

$$(16) \quad -\frac{dS}{dt} = q_s \cdot X$$

$Y_{X/S}$  = growth yield

$q_s$  = specific substrate consumption rate

